Tracking of Moving Targets with MIMO Radar

Peter W. Moo and Zhen Ding
Radar Sensing & Exploitation Section
Defence Research and Development Canada
CANADA

Peter.Moo@forces.gc.ca   Jack.Ding@forces.gc.ca

ABSTRACT

A linear array radar can be operated as a multiple-input multiple-output (MIMO) radar or a directed beam radar. A MIMO radar transmits orthogonal waveforms on each array element, which can achieve virtual aperture extension. Compared to a directed beam radar, MIMO radar requires longer integration times to maintain the same energy on target. This results in narrower Doppler bins but increased range-Doppler migration, which decreases probability of detection. This paper compares the tracking performance of MIMO and Directed Beam radar. The comparison explicitly quantifies differences in beamwidth, Doppler bin width, and probability of detection due to range-Doppler migration. Full and partial velocity and acceleration compensation is considered. Single-target track completeness and track accuracy are compared for directed beam radar, MIMO radar with full compensation, MIMO radar with partial compensation, and uncompensated MIMO radar. It is shown that compensation is required to prevent degraded probability of detection and track completeness as target velocity and acceleration increase.

1.0   INTRODUCTION

A linear array radar is traditionally operated as a phased array radar, also known as a directed beam radar, where each array element transmits an identical waveform with perhaps a phase shift to steer the beam. There is now increasing interest in operating a linear array as a multiple-input multiple-output (MIMO) radar, where distinct elements or subarrays transmit different waveforms. If the waveforms are orthogonal, then their returns can be separated from each other at the radar receiver. Previous work has considered omnidirectional search modes and transmit beamsteering on receive [1], [2]. Rabideau [3] conducted a tradeoff analysis for MIMO radar and directed beam radar by minimizing an objective function that describes the relationship between performance and cost. Target tracking resolution for MIMO radar was described with respect to ambiguity functions in [4]. Multiple target tracking for MIMO radar was considered in [5], where target localization performance was analyzed.

In this paper the tracking performance of directed beam radar and MIMO radar are compared by explicitly quantifying beamwidth, Doppler bin width, and range-Doppler migration for both modes. This paper is organized as follows. Section 2.0 specifies relevant results that enable the tracking comparison. Section 3.0 proposes full and partial compensation schemes to mitigate range-Doppler migration. In Section 4.0 the tracking performance of directed beam and MIMO radar modes are compared. Finally, conclusions are presented in Section 5.0.

2.0   PRELIMINARIES

The radar is a linear antenna array with *M* transmit/receive elements and is operated in one of two modes. In Directed Beam mode, each element transmits the same waveform with a phase shift to

steer the beam. In MIMO mode, each element transmits a distinct orthogonal waveform, through the use of orthogonal frequency division multiplexing or time division multiplexing. Ideal orthogonality between waveforms is assumed.

In both modes, the return signal is received on all elements. Also, the radar carries out Doppler processing in both modes. When comparing the modes, the same physical aperture length will be used.

For the purpose of this study, the point spread function of a static target is assumed to be a delta function in range-Doppler space. In this analysis, spread in range and Doppler will be only due to target motion.

The following parameters describe the radar.
- *PRI* is the pulse repetition interval, in seconds
- *PRF=1/PRI* is pulse repetition frequency
- *N* is the number of pulses in the coherent processing interval (CPI)
- *T* is the length of the CPI, in seconds
- λ is the wavelength, in meters
- ρ is the length of the range cell, in meters.

The following parameters describe the target.
- *v* is the target velocity at the start of the CPI, in m/s,
- *a* is the target acceleration (assumed constant) during the CPI, in m/s^2.

The received signal is sampled and divided into a series of range cells. In each range cell, the sampled signal is subject to Doppler processing. In each range-Doppler bin, target detection is carried out by performing threshold detection; that is, the radar must decide between two hypotheses,

H_0: $z = u$,

H_1: $z = w + u$,

where w is the complex target return and u is zero-mean complex Gaussian noise with variance σ^2. The magnitude of the return $|z|$ is compared to a threshold β, which is chosen to satisfy a specified probability of false alarm P_{fa}; that is, $\mathrm{Pr}(|u| > \beta) = P_{fa}$.

Since the noise magnitude has a Rayleigh distribution, the threshold satisfies $\beta = (\ln P_{fa}^{-\sigma2})^{1/2}$. The width of a Doppler bin is given by $\Omega = 1/T$. The distance traveled by the target during the CPI is $\alpha = vT + aT^2/2$.

The goal of this paper is to compare the single-target tracking performance of Directed Beam mode and MIMO mode for targets with high velocity and acceleration. Detection is assumed to be noise-limited. The comparison explicitly accounts for the increased Doppler resolution and range-Doppler migration of MIMO mode that result from increased integration times.

In Directed Beam mode, the radar transmits numerous directional beams to provide surveillance for a region of interest. In MIMO mode, the radar transmits omni-directionally to cover the same region of interest. Because MIMO mode transmits orthogonal waveforms on each element, the transmitter gain is M times less than for Directed Beam mode. In order to maintain the same energy on target, MIMO mode must use a coherent processing interval (CPI) that is M times longer than for Directed Beam mode. Note that the total time required to provide surveillance for the region of interest is identical for the two modes.

For a fixed length array, there are three key performance characteristics which affect tracking performance of directed beam and MIMO modes: beamwidth, Doppler bin width, and probability of detection due to range-Doppler migration. As shown in [6], the beamwidth of Directed Beam mode θ_{dir} and the beamwidth of MIMO mode θ_{MIMO} are identical.

As explained above, the CPI of MIMO mode is M times longer than that of Directed Beam mode. Since Doppler bin width is the inverse of CPI length, the Doppler bin width of Directed Beam mode Ω_{dir} and the Doppler bin width of MIMO mode Ω_{MIMO} satisfy $\Omega_{\text{MIMO}} = \Omega_{\text{dir}} / M$. Smaller Doppler bin width reduces the velocity estimation error of track measurements.

As derived in [7], the probability of detection is given by

$$P_d \;=\; 1 - \prod_{k=1}^{D} \prod_{n=1}^{i_k} \int_0^{\beta} \frac{2x}{\sigma^2} \exp\left(-\frac{x^2 + c(k,n)^2 y^2}{\sigma^2} \right) I_0\left(c(k,n) \frac{2xy}{\sigma^2} \right) dx,$$

where

$$c(k,n) \;=\; \frac{1}{T} \left(\min\left[d(k), t(x_k + n - i_k + 1) \right] - \max\left[d(k-1), t(x_k + n - i_k) \right] \right), \; n = 1, ..., i_k,$$

$$x_k \;=\; \operatorname*{argmax}_{m:t(m) < d(k)} t(m),$$

$$y_k \;=\; \operatorname*{argmax}_{m:t(m) \leq d(k-1)} t(m),$$

$$i_k \;=\; x_k - y_k + 1,$$

$$S \;=\; \begin{cases} \lceil Q_r \rceil, & \text{if } r \leq \lceil Q_r \rceil \rho - \alpha \\ \lceil Q_r \rceil + 1, & \text{otherwise} \end{cases}$$

$$t(m) \;=\; \begin{cases} 0, & m = 0 \\ \frac{-v + \sqrt{v^2 + 2a(m\rho - r)}}{a}, & 1 \leq m \leq S - 1 \\ T, & m = S \end{cases}$$

$$D \;=\; \begin{cases} \lceil Q_d \rceil, & \text{if } f \leq \lceil Q_d \rceil \Omega - \frac{2aT}{\lambda} \\ \lceil Q_d \rceil + 1, & \text{otherwise} \end{cases}$$

$$d(k) \;=\; \begin{cases} 0, & k = 0 \\ \frac{\lambda}{2a}(k\Omega - f), & 1 \leq k \leq D - 1 \\ T, & k = D \end{cases},$$

and where $Q_d = 2\ a\ T^2/\lambda$, $Q_r = \alpha/\rho$ and I_0 is a modified Bessel function of the first kind. Note that probability of detection decreases if SNR decreases in any one range-Doppler bin, or if the total number of range-Doppler bins containing target returns increases. For the example presented in this paper, it will be assumed that the initial range of the target is in the center of a range cell.

3.0   COMPENSATION OF RANGE-DOPPLER MIGRATION

Target velocity and acceleration can cause range-Doppler migration and result in degraded probability of detection. This section considers the compensation of target velocity and acceleration in signal processing, with the aim of reducing range-Doppler migration.

A number of approaches for carrying out compensation of velocity and acceleration have been developed [8], [9]. In this work, a particular compensation method is not implemented. Instead the effect of velocity and acceleration compensation is modeled by specifying a set of compensation values, selecting the compensation value that is closest to the target velocity or acceleration, and computing the residual velocity or acceleration that results from compensation. By carrying out compensation modeling in this way, the residual velocity or acceleration is similar to what would be obtained from the compensation methods described in [8], [9].

It is assumed that the velocity and acceleration are less than a fixed maximum velocity v_{max} and maximum acceleration a_{max}, respectively. A set of uniformly spaced compensation velocities is specified as $\{0, \acute{v}, 2\acute{v}, \ldots, \text{ceil}(v_{max}/\acute{v})\ \acute{v}\}$, where ceil denotes the ceiling function. A set of uniformly spaced compensation acceleration values is given by á $\{0, á, 2\ á, \ldots, \text{ceil}(a_{max}/á)\ á\ \}$.

In this way, the set of velocity compensation values is entirely specified by the velocity step size $\acute{v}$ and the maximum velocity v_{max}, and the set of acceleration compensation values is entirely specified by the acceleration step size á and the maximum acceleration a_{max}. The value from the set of compensation velocities that is closest to the true velocity is chosen and the received signal is compensated for this velocity value. The effective velocity value after compensation v_c is the difference between the target velocity and the closest velocity from the set of compensation velocities. Similarly, the effective acceleration value after compensation a_c is the difference between the target acceleration and the closest acceleration from the set of compensation acceleration values.

In the following, the velocity and acceleration step sizes are specified for full and partial compensation.

3.1 Full compensation

For full velocity compensation, the velocity step size is given by $\acute{v}_{opt} = \rho/T$. With this choice of velocity step size, max $|v_c| = \rho/2T$. The resulting residual velocity after compensation limits range migration effects.

Full acceleration compensation has an acceleration step size given by $á_{opt} = \lambda/2T^2$. With this choice of acceleration step size, max $|a_c| = \lambda/4T^2$. After compensation, the effective acceleration limits Doppler migration effects.

For most types of compensation, the received signal must be processed with all possible compensation values. Therefore, the use of full compensation can lead to high computational complexity, especially if the sets of compensation values are large, such as when the maximum velocity and acceleration values are large. This motivates the consideration of partial compensation, where the step sizes are greater.

3.2 Partial compensation

Partial compensation uses larger step sizes than full compensation. As a result, the sets of compensation values are smaller, which reduces computational complexity. In this study, partial compensation step sizes are expressed relative to full compensation step sizes. Specifically, $ú_\mu = ú_{opt}/\mu$, and $á_\nu = á_{opt}/\nu$. where $0 < \mu \leq 1$ and $0 < \nu \leq 1$. For partial compensation with $\mu = 0.5$ and $\nu = 0.5$, the spacing between compensation values is twice that of full compensation. When $\mu = 1$ and $\nu = 1$, partial compensation is identical to full compensation. The degradation in performance when using partial compensation compared to full compensation will vary with radar and target parameters.

4.0   TRACKING COMPARISON

In this section, the tracking performance of MIMO mode is compared to that of Directed Beam mode. Compared to Directed Beam mode, target detections in MIMO mode will have improved range rate estimation accuracy due to smaller Doppler bin width. However, MIMO mode may suffer from degraded probability of detection due to range-Doppler migration, unless velocity and acceleration compensation is implemented. The effect of these characteristics on tracking performance will be considered by examining an S-band tracking scenario for four cases:
   1. Directed Beam mode
   2. MIMO mode with full velocity or acceleration compensation
   3. MIMO mode with partial velocity or acceleration compensation
   4. MIMO mode without compensation

Details of the two-dimensional tracking scenario are as follows. The radar is a 16-element linear array with a length of 2 m and operates at a wavelength of 0.15 m, corresponding to a frequency of 2 GHz. The range cell length is 20 m. A target with a constant RCS is assumed. Although not considered here, it is possible to use a Swerling I or III target model, and it is expected that the results would be similar to those for a constant RCS target. The target has an initial range of 100 km from the radar and is located at zero degrees azimuth relative to the center of the linear array. At the initial range of 100 km, the target SNR is 19 dB. The target travels towards the radar for 90 seconds at zero degrees azimuth, with an initial velocity v and a constant acceleration a. For velocity and acceleration compensation, the maximum velocity is $v_{max} = 250\$$ m/s and the maximum acceleration is $a_{max} = 1.0$ m/s^2. The probability of false alarm is 10^{-5}. For Directed Beam mode, the beamwidth is 3.78 degrees, and the Doppler bin width is 20 Hz. T the beamwidth in MIMO mode is also 3.78 degrees, while the Doppler bin width is $20/16 = 1.25$ Hz. An update interval of two seconds is used for both MIMO mode and Directed Beam mode.

Starting at time zero, the radar attempts to detect the target at an update interval of two seconds with probability P_d, which is evaluated using the expression from Section 2.0. The target SNR increases as the target travels towards the radar. If the target is detected, a target measurement vector is generated by adding Gaussian noise to the ground truth. A target measurement vector consists of a range measurement, an azimuth measurement, and a range rate measurement.

The tracker employs an Interacting Multiple Model algorithm [10] that incorporates a constant velocity model and a Singer maneuvering model for estimating target dynamics. As target behavior is not known a priori, it is difficult to predict its behavior based on a single maneuver model. The IMM algorithm is a robust technique that combines two hypothesized models according to a Markov model for the transition from one target maneuver model to another. For the target acceleration values under consideration, the constant velocity model consistently had a higher model probability.

For each scenario where *v* and *a* are specified, 500 Monte Carlo runs were generated. At each time instant, the resulting track completeness and track accuracy were averaged over all runs for which a track existed.

For the simulation results, the target acceleration was 0.1 m/s^2, and the target velocity was varied from 12.5 m/s to 250 m/s in increments of 12.5 m/s. The target acceleration value was chosen to be small enough so that target acceleration would not cause Doppler migration. For the scenario parameters, the velocity step size for full compensation is calculated as $\acute{\upsilon}_{opt}$ =25 m/s so that the set of full compensation velocities is {0, 25, 50, 75, 100, 125, 150, 175, 200, 225, 250} m/s.

MIMO with partial compensation used partial velocity compensation with μ =0.2 and no acceleration compensation. The partial velocity compensation step size is $\acute{\upsilon}_{0.2 =}$ = 125 m/s, and the set of partial compensation velocities is {0, 125, 250} m/s.

Figures 1 to 3 present the detection and tracking results. In Figure 1, the probability of detection at the initial target range of 100 km is shown. Directed Beam mode has a probability of detection of one for all velocity values. For uncompensated MIMO, probability of detection is 0.2 or less for velocity values of 100 m/s and greater. For MIMO with full compensation, the effective target velocity after compensation is 0 m/s or 12.5 m/s. The probability of detection for MIMO with full compensation is one for all velocity values. For MIMO with partial compensation, the effective velocity after compensation is the difference between the true velocity and the nearest compensation velocity. It is seen that probability of detection varies with effective velocity, from a high of 1 when the effective velocity is zero to a low of 0.66 when the magnitude of the effective velocity is 62.5 m/s. Probability of detection will increase as the target moves towards the radar, due to decreasing range. Since the scenario time is fixed at 90 seconds, as velocity increases, the target range at the end of the scenario will decrease. The slight decrease in probability of detection at 25 m/s, 100 m/s, 150 m/s, and 225 m/s is caused by the variation in initial modulus Doppler frequency for these velocity values.
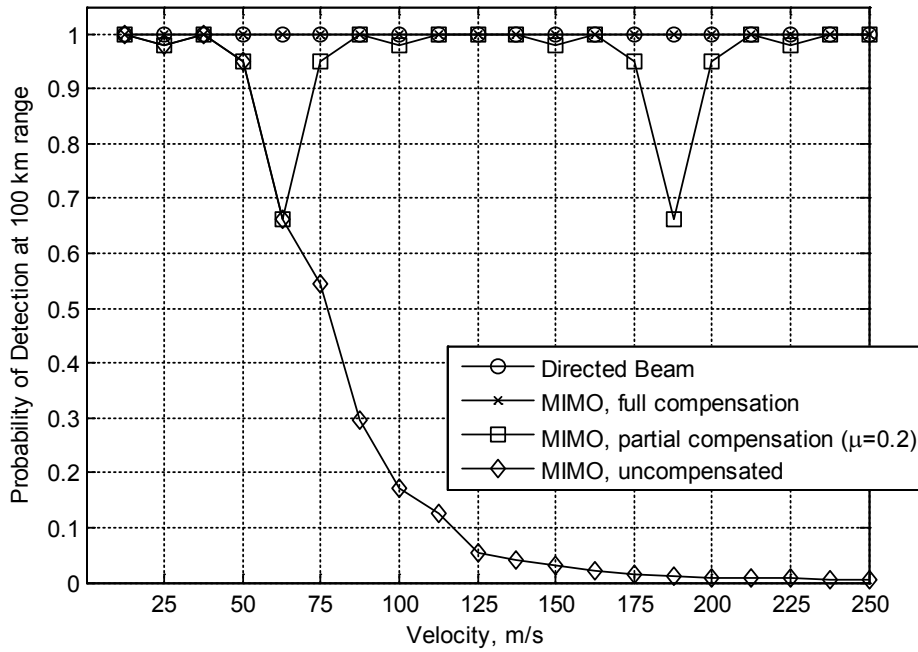
Figure 1: S-band scenario with target acceleration of 0.1 m/s^2: probability of detection at 100 km range.
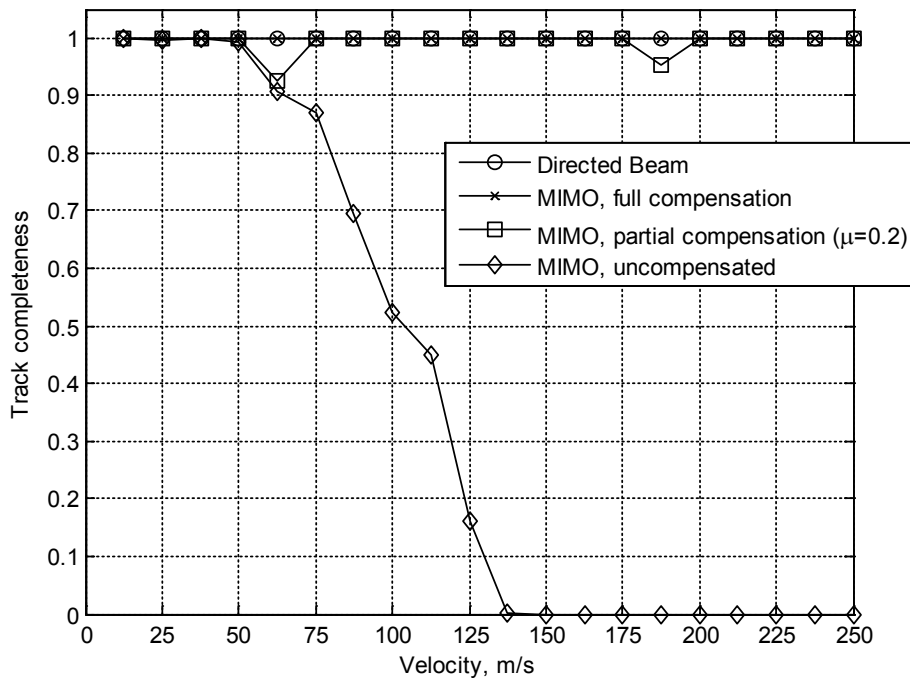

Figure 2: S-band scenario with target acceleration of 0.1 m/s^2: average track completeness.

Figure 2 shows track completeness for all four cases. Directed Beam and MIMO with full compensation have track completeness values of essentially one. For both cases, the target is detected at almost every update interval, which updates the track. For MIMO with partial compensation and uncompensated MIMO, track completeness is greater than probability of detection at the initial range. This is mostly due to the ability of the tracker to coast over missed measurements. That is, even when several updates do not produce a measurement, it still may be possible for the tracker to predict a track based on the track history. The increased track completeness relative to probability of detection at the initial range is also partly due to the increase in probability of detection as target range decreases. This effect can be seen in the MIMO with partial compensation case for target velocities of 62.5 m/s and 187.5 m/s. For these velocities, the initial probability of detection is 0.66, but track completeness is slightly larger for the higher velocity target.
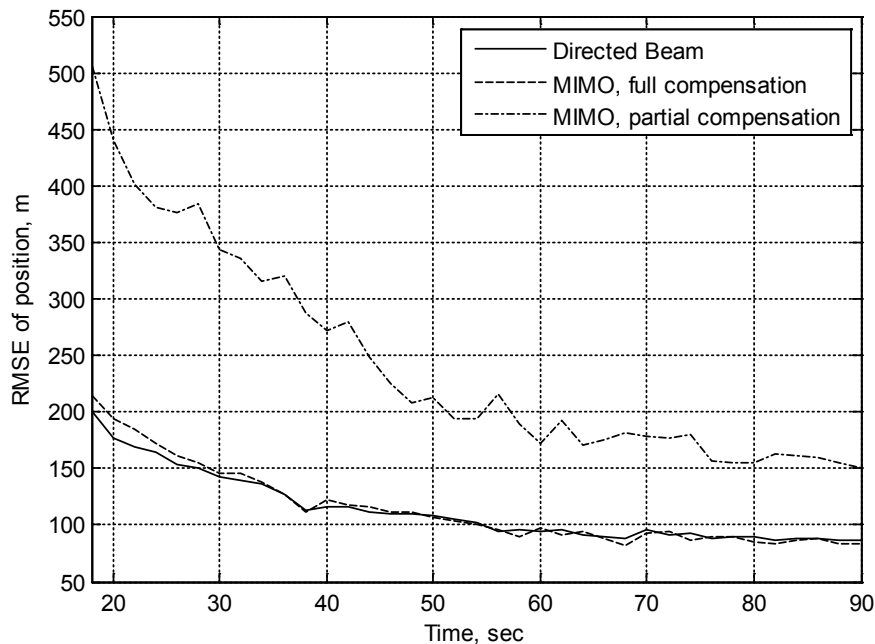

Figure 3: S-band scenario with target acceleration of 0.1 m/s^2: position RMSE for a target velocity of 187.5 m/s.

Figure 3 shows position root mean-squared error (RMSE) for Directed Beam, MIMO with full compensation, and MIMO with partial compensation for a target velocity of 187.5 m/s. Directed Beam and MIMO with full compensation have essentially the same RMSE. The two modes have probability of detection and track completeness that are essentially one. Although MIMO with full compensation has smaller Doppler bin width, and therefore improved velocity (i.e. range rate) estimation accuracy, this has no effect on RMSE. MIMO with partial compensation has larger RMSE than the other two cases. MIMO with full compensation and MIMO with partial compensation have the same velocity estimation accuracy. However, MIMO with partial compensation has smaller

probability of detection and coasts over a number of missed measurements. These missed measurements result in increased position RMSE.

5.0   CONCLUSIONS

For a linear array radar, the tracking performance of MIMO mode was compared to that of Directed Beam mode for targets with varying values of velocity and acceleration. MIMO mode requires longer integration times compared to Directed Beam mode which can cause target returns to be spread over multiple range-Doppler bins. Step sizes for velocity and acceleration compensation were proposed. Full compensation mitigates the effects of range-Doppler migration but has significant computational complexity. Partial compensation has reduced computational complexity but may not completely eliminate the effects of range-Doppler migration.

Single-target tracking performance was analyzed for Directed Beam mode, MIMO mode with full compensation, MIMO mode with partial compensation, and uncompensated MIMO mode. For the example considered, results showed that Directed Beam mode and MIMO with full compensation achieved probability of detection and track completeness close to one. MIMO mode with partial compensation had reduced computational complexity compared to fully compensated MIMO, but suffered from degraded tracking performance due to missed detections which forced the tracker to coast. Uncompensated MIMO mode had low track completeness as velocity and acceleration increased. This poor performance was caused by range-Doppler migration resulting from longer integration times.

6.0   REFERENCES

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